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Fuzzy Partial Ordering

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ABSTRACT

In the present paper, introduced a fuzzy partial order and weakly order using fuzzy metric space and fuzzy partial metric space.

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INTRODUCTION

The concepts of fuzzy sets operations were first introduced by L.A.Zadeh in his classical paper in the year 1965. Thereafter the paper of the C.L.Chang in 1968 paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Since then much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed. The notion of continuity is of fundamental importance in almost all branches of mathematics. Hence it is of considerable signification from applications view point, to formulate and study new variants of fuzzy partial ordering.

Preliminaries

A weakly order consistent fuzzy topology is a weaker version of the order consistent fuzzy topology as used in fuzzy lattice theory for which in addition supreme of directed sets are their limits. An intersecting example of a weakly order consistent fuzzy topology is the fuzzy topology of all upward closed sets, $T[\leq] = \{S \subseteq X / \forall x \in S, \quad x \leq y \Rightarrow y \in S\}$.

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Definition 1: a basis A for a fuzzy topology is fuzzy σ disjoint, if there A1, A2... $\subseteq$ A such that $A = \bigcup\{B_n / n \in \omega\}$ $\forall n \in \omega, n \in B \quad B' \in B_n, B \cap B' = \phi$ **.**

Definition 2: A fuzzy partial ordering in fuzzy topological space (X, T) is a binary relation $\sqsubseteq\ \subseteq X \times X$ such that

(PO1) $\forall x \in X, \quad x \sqsubseteq x$

(PO2) $\forall x, y \in X, \quad x \sqsubseteq y \cap y \sqsubseteq x \Rightarrow x = y$

(PO3) $\forall x, y, z \in X, \quad x \sqsubseteq y \cap y \sqsubseteq z \Rightarrow x \sqsubseteq z$

Definition 3: A fuzzy point P in fuzzy topological space (X, T) is a special fuzzy set with membership function defined by $P(x) = \begin{cases} 1 & x = y \\ 0 & x \neq y \end{cases}$, where $0 < a < 1$ is said to have support y, value λ and is denoted by p_y^a or P(y, λ).

Definition 4: A fuzzy partial metric on a nonempty set X is a function $d : X \times X \to [0, \infty)$ such that for all x, y, z $\in$ X:

(P1) $x = y \Leftrightarrow d(p_x^a, p_x^a) = d(p_x^a, p_y^b) = d(p_y^b, p_y^b)$

(P2) $d(p_x^a, p_x^a) \leq d(p_x^a, p_y^b)$

(P3) $d(p_x^a, p_y^b) = d(p_y^b, p_x^a)$

(P4) $d(p_x^a, p_y^b) \leq d(p_x^a, p_z^c) + d(p_z^c, p_y^b) - d(p_z^c, p_z^c)$

A fuzzy partial metric space is a pair (X, d) such that X is a nonempty set and d is a fuzzy partial metric on X.

The fuzzy partial metric axioms (P1) and (P4) are intended to be a minimal generalization of the metric axioms (M1) and (M3) such that each object does not necessarily have to have zero distance from itself. In this generalization we manage to preserve the symmetry axiom (M2) to get (P3), but have to massage the transitivity axiom (M3) to produce the generalization (P4).

Example: The pair (R^+, d), where $d(p_x^a, p_y^b) = \mathrm{Max}\{x, y\}$ for all $x, y \in R^+$.

Definition 5: An open ball for a fuzzy partial metric $d : X \times X \to [0, \infty)$ is a set of the form, $B_\varepsilon^d(x) = \{y \in X / d(p_x^a, p_y^b) < \varepsilon\}$ for each $\varepsilon > 0$ and $x \in X$. Note that, unlike their metric counterparts, some fuzzy partial metric open balls may be empty. For example, if $d(p_x^a, p_x^a) > 0$, then $B^d_{d(p_x^a, p_x^a)}(x) = \phi$.

Definition 6: Fuzzy topological space (X, T) is called a fuzzy T_0 space if and only if for any fuzzy points x and y such that $x \neq y$, either $x \notin \bar{y}$ or $y \notin \bar{x}$

Proposition 1: Each fuzzy topology T_0 is weakly order consistent if and only if it is a fuzzy topology of upwardly closed sets

Theorem 1: Each fuzzy partial metric is fuzzy topology T_0.

Proof:

Suppose $d : X \times X \to [0,\infty)$ is fuzzy partial metric and suppose $x \neq y \in X$, then, from P1 &P2 which implies $d(p_x^a, p_x^a) \leq d(p_x^a, p_y^b)$ and so $x \in B_\varepsilon^d(x) \wedge y \neq B_\varepsilon^d(x)$ where $\varepsilon = (d(p_x^a, p_x^a) + d(p_x^a, p_y^b))/2$.

Proposition 2: Each fuzzy partial metric is weakly order consistent if and only if it is a fuzzy topology of upwardly closed sets

Definition 7: let (X, T) be fuzzy topological space, for each fuzzy partial metric $d : X \times X \to [0,\infty)$, $\sqsubseteq_d \subseteq X \times X$ is the binary relation such that

$$\forall x, y \in X, x \sqsubseteq_d y \Leftrightarrow d(p_x^a, p_x^a) = d(p_x^a, p_y^y)$$

Theorem 2: for each fuzzy partially metric d, $\sqsubseteq_d$ is a fuzzy partial ordering.

Proof: we prove three conditions as follows

(PO1) $\forall x \in X, \ \ x \sqsubseteq_d x \ \ \text{as} \ \ d(p_x^a, p_x^a) = d(p_x^a, p_x^a)$

(PO2)
$$\begin{aligned}
&\forall x, y \in X, \ \ x \sqsubseteq_d y \wedge y \sqsubseteq_d x \\
&\Rightarrow d(p_x^a, p_x^a) = d(p_x^a, p_y^b) = d(p_y^b, p_y^b) \quad \text{by}(P3) \\
&\Rightarrow x = y \quad \text{by}(P1)
\end{aligned}$$

(PO3)
$$\begin{aligned}
&\forall x, y, z \in X, \ \ x \sqsubseteq_d y \wedge y \sqsubseteq_d z \\
&\Rightarrow d(p_x^a, p_x^a) = d(p_x^a, p_y^b) \wedge d(p_y^b, p_y^b) = d(p_y^b, p_z^c) \ \text{by}(P3) \\
&\Rightarrow \text{but by } (P4), d(p_x^a, p_z^c) \leq d(p_x^a, p_y^b) + d(p_y^b, p_z^c) - d(p_y^b, p_y^b) \\
&\therefore d(p_x^a, p_z^c) \leq d(p_x^a, p_x^a) \\
&\therefore d(p_x^a, p_z^c) = d(p_x^a, p_x^a) \\
&x \sqsubseteq_d z
\end{aligned}$$

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