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Fuzzy Weakly Ordering

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ABSTRACT

In the present paper, introduced a fuzzy weakly ordering using fuzzy metric space and fuzzy partial metric space.

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INTRODUCTION

In order to deal with uncertainties, the idea of fuzzy sets and fuzzy set operations was introduced by L.A.Zadeh in his classical paper in the year 1965. This inspired mathematicians to fuzzily mathematical structures. The first notion of fuzzy topological space had been defined by C.L.Chang in 1968. S.G.Matthews introduced in his paper the partial metric topology were discussed partial metric, quasi metric, partial ordering and etc. in this paper I motivated above topics into fuzzy topological space.

Preliminaries

Any fuzzy topology T0 the information ordering can be recovered using the specialization ordering defined by $x \le y \Rightarrow x \in cl(\{y\})$, we are interested in fuzzy totally ordered sequences $A \in X^{\omega}$ of the form $A_0 \le A_1 \le A_2 \le ....$ called fuzzy chains of increasing information, the least upper bound lub(A) of which is intended to capture the notion of the amount of information defined by the fuzzy chain. To ensure that lub(A) cannot contain more information than can be derived from the members of the fuzzy chain X we insist that our fuzzy topology have the following property. An

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intersecting example of a weakly order consistent fuzzy topology is the fuzzy topology of all upward closed sets, $T[\sqsubseteq] = \{S \subseteq X / \forall x \in S, \quad x \sqsubseteq y \Rightarrow y \in S\}$.

Definition 1: A fuzzy point P in fuzzy topological space (X, T) is a special fuzzy set with membership function defined by $P(x) = \begin{cases} 1 & x = y \\ 0 & x \neq y \end{cases}$, where $0 < a < 1$ is said to have support y, value λ and is denoted by P_y^a or P(y, λ).

Definition 2: A scott like fuzzy topology over a fuzzy partial ordering $\sqsubseteq \subseteq X \times X$ is a fuzzy weakly ordering consistent fuzzy topology T over X such that for each fuzzy chain $A \in X^{\omega}$, lub(A) exists, and $\forall o \in T, \mathrm{lub}(A) \in o \Rightarrow \exists k \in \omega \quad \forall n > k, A_n \in o$.

Definition 3: A fuzzy partial metric on a nonempty set X is a function $d : X \times X \to [0, \infty)$ such that for all x, y, z $\in$X:

(P1) $x = y \Leftrightarrow d(p_x^a, p_x^a) = d(p_x^a, p_y^b) = d(p_y^b, p_y^b)$

(P2) $d(p_x^a, p_x^a) \leq d(p_x^a, p_y^b)$

(P3) $d(p_x^a, p_y^b) = d(p_y^b, p_x^a)$

(P4) $d(p_x^a, p_y^b) \leq d(p_x^a, p_z^c) + d(p_z^c, p_y^b) - d(p_z^c, p_z^c)$

A fuzzy partial metric space is a pair (X, d) such that X is a nonempty set and d is a fuzzy partial metric on X.

The fuzzy partial metric axioms (P1) and (P4) are intended to be a minimal generalization of the metric axioms (M1) and (M3) such that each object does not necessarily have to have zero distance from itself. In this generalization we manage to preserve the symmetry axiom (M2) to get (P3), but have to massage the transitivity axiom (M3) to produce the generalization (P4).

NB: T[p] denoted by fuzzy partial metric topology.

Example: The pair (R^+, d), where $d(p_x^a, p_y^b) = \mathrm{Max}\{x, y\}$ for all $x, y \in R^+$.

Definition 4: A fuzzy quasi metric is a function $d : X \times X \to [0, \infty)$ such that,

(Q1) $\forall x, y \in X, x = y \Leftrightarrow d(q_x^a, q_y^b) = d(q_y^b, q_x^a) = 0$

(Q2) $\forall x, y, z \in X, d(q_x^a, q_z^c) \leq d(q_x^a, q_y^b) + d(q_y^b, q_z^c)$

NB: T[q] denoted by fuzzy quasi metric topology.

Lemma 1: for each fuzzy quasi metric $d : X \times X \to [0, \infty)$ the relation $\sqsubseteq_d \subseteq X \times X$ defined by $\forall x, y \in X, x \leq_d y \Leftrightarrow d(q_x^a, q_y^b) = 0$

Lemma 2: for each fuzzy quasi metric $d: X\times X \to [0,\infty)$ the set of all open balls of the form, $B_{\varepsilon}^{d} = \{y \in X / d(q_x^a, q_y^b) < \varepsilon\}$ for each $\forall x \in X$ and $\varepsilon > 0$ is the basis for a fuzzy weakly consistent topology T[q] over $\preceq_d$.

Lemma 3: for each fuzzy quasi metric $d: X\times X \to [0,\infty)$ the symmetrisation function $d^S: X\times X \to [0,\infty)$ for q where $\forall x, y \in X, d(qS_x^a, qS_y^b) \Leftrightarrow d(q_x^a, q_y^b) + d(q_y^b, q_x^a)$. Is a fuzzy metric such that $T[q] \subseteq T[q^S]$

Lemma 4: (the fuzzy quasi metric contraction mapping theorem) for each fuzzy quasi metric $d: X\times X \to [0,\infty)$ such that qS is fuzzy complete, and for each function $f: X \to X$ such that $\exists 0 \le c < 1, x, y \in X, d(q_{f(x)}^a, q_{f(y)}^b) \le c \times d(q_x^a, q_y^b)$ called a contraction, there exists a unique $n \in X$ such that $n = f(n)$ and $\varepsilon > 0$ is the basis for a fuzzy weakly consistent topology T[q] over $\preceq_d$

Definition 5: A fuzzy partial ordering in fuzzy topological space (X, T) is a binary relation $\preceq \subseteq X \times X$ such that

(PO1) $\forall x \in X, \quad x \preceq x$

(PO2) $\forall x, y \in X, \quad x \preceq y \cap y \preceq x \Rightarrow x = y$

(PO3) $\forall x, y, z \in X, \quad x \preceq y \cap y \preceq z \Rightarrow x \preceq z$

Definition 6: An open ball for a fuzzy partial metric $d: X\times X \to [0,\infty)$ is a set of the form, $B_{\varepsilon}^{d}(x) = \{y \in X / d(p_x^a, p_y^b) < \varepsilon\}$ for each $\varepsilon > 0$ and $x \in X$. Note that, unlike their metric counterparts, some fuzzy partial metric open balls may be empty. For example, if $d(p_x^a, p_x^a) > 0$, then $B_{d(p_x^a, p_x^a)}^{d}(x) = \phi$.

Definition 7: Fuzzy topological space (X, T) is called a fuzzy T0 space if and only if for any fuzzy points x and y such that $x \ne y$, either $x \notin \bar{y}$ or $y \notin \bar{x}$

Definition 8: let (X, T) be fuzzy topological space, for each fuzzy partial metric $d: X\times X \to [0,\infty)$, $\preceq_d \subseteq X \times X$ is the binary relation such that $\forall x, y \in X, x \preceq_d y \Leftrightarrow d(p_x^a, p_x^a) = d(p_x^a, p_y^y)$

Theorem 1: for each fuzzy partially metric d, $\preceq_d$ is a fuzzy partial ordering.

Proof: we prove three conditions as follows

(PO1) $\forall x \in X, \quad x \preceq_d x \quad \text{as} \quad d(p_x^a, p_x^a) = d(p_x^a, p_x^a)$

$$\forall x, y \in X, \quad x \sqsubseteq_d y \wedge y \sqsubseteq_d x$$
$$\Rightarrow d(p_x^a, p_x^a) = d(p_x^a, p_y^b) = d(p_y^b, p_y^b) \quad \text{by}(P3)$$
$$\Rightarrow x = y \quad \text{by}(P1)$$

(PO2)

$$\forall x, y, z \in X, \quad x \sqsubseteq_d y \wedge y \sqsubseteq_d z$$
$$\Rightarrow d(p_x^a, p_x^a) = d(p_x^a, p_y^b) \wedge d(p_y^b, p_y^b) = d(p_y^b, p_z^c) \ \text{by}(P3)$$
$$\Rightarrow \text{but by } (P4), d(p_x^a, p_z^c) \leq d(p_x^a, p_y^b) + d(p_y^b, p_z^c) - d(p_y^b, p_y^b)$$
$$\therefore d(p_x^a, p_z^c) \leq d(p_x^a, p_x^a)$$
$$\therefore d(p_x^a, p_z^c) = d(p_x^a, p_x^a)$$
$$x \sqsubseteq_d z$$

(PO3)

Theorem 2: for each fuzzy partial metric, $T[p] \subseteq T[\sqsubseteq_d]$

Proof: it is sufficient to show that,

$$\forall x \in X, \forall \varepsilon > 0. B_\varepsilon^d(x) = \cup\{\{z / y \sqsubseteq_d z\} / y \in B_\varepsilon^d(x)\}$$

suppose $x, y, z \in X$ and $\varepsilon > 0$ are such that $y \in B_\varepsilon^d(x)$ and $y \sqsubseteq_d z$ Then

$$d(p_x^a, p_z^c) \leq d(p_x^a, p_y^b) + d(p_y^b, p_z^c) - d(p_y^b, p_y^b) \quad \text{by } (P4)$$

$$= d(p_x^a, p_y^b) \quad \text{as } y \sqsubseteq_d z$$
$$< \varepsilon \quad \text{as } y \in B_\varepsilon^d(x)$$

Thus $Z \in B_\varepsilon^d(x)$

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