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Fuzzy Partial Metric Spaces

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ABSTRACT

In this research study explain the introduced of fuzzy partial metric space using fuzzy metric space and partial metric space. This paper was given to the finding and suggestions.

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INTRODUCTION

In the history of science, new theories have always been necessary in order for existing scientific theories to process and this will continue to be true in the future. The theories of fuzzy set were introduced by Zadehi in this paper traditional. This theory provides a natural foundation for treating mathematically the fuzzy observable fact, which exist pervasively in our real world, and for building new branches of fuzzy mathematics. Using the concept of fuzzy sets, Chang [2] first introduced fuzzy topological space. After that many authors have applied various basic concept of general topology to fuzzy set, and developed theories of fuzzy topological space.

Preliminaries

A fuzzy set A in a space set X is categorization by a membership(characteristic) purpose of $\mu_A : X \to I$, which an associated with each point in X a real number in closed interval I = [0, 1]. The collection of all fuzzy subset in X will be denoted by IX. A family

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Ton fuzzy set X is called a fuzzy topology on X if ϕ and X belong to T and T is closed with respect to arbitrary union and finite intersection. The member of T is called fuzzy open sets and their complements are fuzzy closed sets. In this paper we use the notation fuzzy topology in the original sense of Chang [1].We shall denote a fuzzy topological space (fts. For short) by (X, T), where X is the underlying set and T is the fuzzy topology. The symbols $\mu, \delta, \lambda, \ldots$ etc. are used to denote fuzzy sets and the symbols $\mu(x), \delta(x), \lambda(x), \ldots$ etc. are used to denote the membership function for this sets, the symbol (1-λ) stands for the complement of the fuzzy set.

In the definition of Partial metric space is motivated in the name of fuzzy partial metric space as follows, in this paper some result on fuzzy partial metric space shall be discussed and it is an fuzzy T0.

Definition 1: A fuzzy point P in fuzzy topological space (X, T) is a special fuzzy set with membership function defined by $P(x) = \begin{cases} 1 & x = y \\ 0 & x \neq y \end{cases}$, where $0 < a < 1$ is said to have support y, value λ and is denoted by P_y^a or P(y, λ).
NB:

1. It complement of the fuzzy point P_y^a is denoted by P_y^{1-a}.
2. P_y^1 is called crops point

Definition 2: A fuzzy partial metric on a nonempty set X is a function $d : X \times X \to [0, \infty)$ such that for all x, y, z $\in$ X:

(P1) $x = y \Leftrightarrow d(p_x^a, p_x^a) = d(p_x^a, p_y^b) = d(p_y^b, p_y^b)$

(P2) $d(p_x^a, p_x^a) \leq d(p_x^a, p_y^b)$

(P3) $d(p_x^a, p_y^b) = d(p_y^b, p_x^a)$

(P4) $d(p_x^a, p_y^b) \leq d(p_x^a, p_z^c) + d(p_z^c, p_y^b) - d(p_z^c, p_z^c)$

A fuzzy partial metric space is a pair (X, d) such that X is a nonempty set and d is a fuzzy partial metric on X.

The fuzzy partial metric axioms (P1) and (P4) are intended to be a minimal generalization of the metric axioms (M1) and (M3) such that each object does not necessarily have to have zero distance from itself. In this generalization we manage to preserve the symmetry axiom (M2) to get (P3), but have to massage the transitivity axiom (M3) to produce the generalization (P4).

Example: The pair (R^+, d), where $d(p_x^a, p_y^b) = Max\{x, y\}$ for all $x, y \in R^+$.

Definition 3: An open ball for a fuzzy partial metric $d : X \times X \to [0, \infty)$ is a set of the form, $B_\varepsilon^d(x) = \{y \in X / d(p_x^a, p_y^b) < \varepsilon\}$ for each $\varepsilon > 0$ and $x \in X$. Note that, unlike their metric counterparts, some fuzzy partial metric open balls may be empty. For example, if $d(p_x^a, p_x^a) > 0$, then $B_{d(p_x^a, p_x^a)}^d(x) = \phi$.

Theorem 1: The set of all open balls of a fuzzy partial metric $d : X \times X \to [0, \infty)$ is the fuzzy topological space (X, T).

Proof:

As, $X = \bigcup_{x \in X} B_{d(p_x^a, p_x^a)+1}^d(x)$ and for any balls $B_\varepsilon^d(x)$ and $B_\delta^d(y)$

$$B_\varepsilon^d(x) \cap B_\delta^d(y) = \bigcup\{B_\eta^d(z) / z \in B_\varepsilon^d(x) \cap B_\delta^d(y)\}$$

Where, $\eta = d(p_z^c, p_z^c) + \min\{\varepsilon - d(p_x^a, p_z^c), \delta - d(p_y^b, p_z^c)\}$.

Theorem 2: For each fuzzy partial metric d, open ball $B_\varepsilon^d(a)$, and $x \in B_\varepsilon^d(a)$, there exists $\delta > 0$ such that $x \in B_\delta^d(x) \subseteq B_\varepsilon^d(a)$.

Proof:

Suppose $x \in B_\varepsilon^d(a) \Rightarrow d(p_x^a, p_a^{a'}) < \varepsilon$

Let $\delta = \varepsilon - d(p_x^a, p_a^{a'}) + d(p_x^a, p_x^a) \Rightarrow \delta > 0 \text{ as } \varepsilon > d(p_x^a, p_a^{a'})$

Also $d(p_x^a, p_x^a) < \delta \text{ as } \varepsilon > d(p_x^a, p_a^{a'})$

Thus $x \in B_\delta^d(x)$

Suppose now that $y \in B_\delta^d(x)$

$$\therefore d(p_y^b, p_x^a) < \delta$$

$$\therefore d(p_y^b, p_x^a) < \varepsilon - d(p_x^a, p_a^{a'}) + d(p_x^a, p_x^a)$$

$$\therefore d(p_y^b, p_x^a) + d(p_x^a, p_a^{a'}) - d(p_x^a, p_x^a) < \varepsilon$$

$$\therefore d(p_y^b, p_x^a) < \varepsilon \quad \text{by (P4)}$$

$\therefore y \in B_{\varepsilon}^{d}(x)$

Thus $B_{\delta}^{d}(x) \subseteq B_{\varepsilon}^{d}(a)$.

NB

Using the last result it can be shown that each sequence $A \in X^{\omega}$ convergence to an object $a \in X$

if and only if $\lim_{n \to \infty} d(p_{A_n}^{a}, p_{a}^{a'}) = d(p_{a}^{a'}, p_{a}^{a'})$.

Definition 4: Fuzzy topological space (X,T) is called a fuzzy T0 space if and only if for any fuzzy points x and y such that $x \neq y$, either $x \notin \bar{y}$ or $y \notin \bar{x}$

Theorem 3: Each fuzzy partial metric is fuzzy T0.

Proof:

Suppose $d : X \times X \to [0,\infty)$ is fuzzy partial metric and suppose $x \neq y \in X$, then, from P1 &P2 which implies $d(p_{x}^{a}, p_{x}^{a}) \leq d(p_{x}^{a}, p_{y}^{b})$ and so $x \in B_{\varepsilon}^{d}(x) \wedge y \neq B_{\varepsilon}^{d}(x)$ where $\varepsilon = (d(p_{x}^{a}, p_{x}^{a}) + d(p_{x}^{a}, p_{y}^{b}))/2$.

References

[1]C. L CHANG, Fuzzy Topological Spaces, J. Math. Anal. Appl. , 24, (1968),182-190

[2]Partial Metric Topology, S.G. Matthews, in, Papers on General Topology and Applications, Eighth Summer Conference at Queens College. Eds. S. Andimaet.al. Annals of the New York Academy of Sciences, vol. 728, pp. 183-197.

[3]S.G. Matthews, Partial metric topology, Research Report 212, Dept. of Computer Science, University of Warwick, 1992.

[4]I.Altun,F.Sola,H Simsek: generalized contractions on partial metric spaces: topology and its applications 157 (2010) 2778-2785.

[i] L. A. ZADEH, Fuzzy Sets, Information and Control, Vol. 8, 338-358, 1965.