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TRIPLE CONNECTED COMPLEMENTARY TREE DOMINATION NUMBER OF A GRAPH

V. MURUGAN
Department of Mathematics
Sri Vidya Mandir Arts & Science College
Uthangarai, Krishnagiri (DT)-636902, T.N. India

V.GOVINDAN
Department of Mathematics
Sri Vidya Mandir Arts & Science College
Uthangarai, Krishnagiri (DT)-636902, T.N. India

K.PARVATHI
Department of Mathematics
Sri Vidya Mandir Arts & Science College
Uthangarai, Krishnagiri (DT)-636902, T.N. India

Abstract

The concept of triple connected graphs with real life application was introduced in [14] by considering the existence of a path containing any three vertices of a graph G. In [4], G. Mahadevan et. al., introduced triple connected domination number of a graph. A subset *S* of *V* of a nontrivial connected graph *G* is said to be triple connected dominating set, if *S* is a dominating set and the induced sub graph <*S*> is triple connected. The minimum cardinality taken over all triple connected dominating sets is called the triple connected domination number of G and is denoted by $_{tc}(G)$. A subset *S* of *V* of a nontrivial graph connected graph *G* is said to be a complementary tree dominating set, if *S* is a dominating set and the induced sub graph <*V* - *S*> is a tree. The minimum cardinality taken over all complementary tree dominating sets is called the complementary tree domination number of G and is denoted by $_{ctd}(G)$. In this paper we introduce a new domination parameter, called triple connected complementary tree domination number of a graph. A subset *S* of *V* of a nontrivial connected graph *G* is said to be triple connected complementary tree dominating set, if *S* is a triple connected dominating set and the induced sub graph <*V* - *S*> is a tree. The minimum cardinality taken over all triple connected complementary tree dominating sets is called the triple connected complementary tree domination number of G and is denoted by

$_{tct}(G)$. We determine this number for some standard graphs and obtain bounds for general graphs. Its relationship with other graph theoretical parameters are also investigated.

Mathematics Subject Classification: 05C69

Keywords: Domination Number, Triple connected graph, Triple connected domination number, Triple connected complementary tree domination number.

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1. Introduction

By a ***graph*** we mean a finite, simple, connected and undirected graph $G(V, E)$, where V denotes its vertex set and E its edge set. Unless otherwise stated, the graph G has p vertices and q edges. ***Degree*** of a vertex v is denoted by $d(v)$, the ***maximum degree*** of a graph G is denoted by $\Delta(G)$. We denote a ***cycle*** on $p$vertices by C_p, a ***path*** on p vertices by P_p, and a ***complete graph*** on p vertices by K_p. A graph G is ***connected*** if any two vertices of G are connected by a path. A maximal connected subgraph of a graph G is called a ***component*** of G. The number of components of G is denoted by (G). The ***complement*** $\bar{}$ of G is the graph with vertex set V in which two vertices are adjacent if and only if they are not adjacent in G. A ***tree*** is a connected acyclic graph. A ***bipartite graph*** (or ***bigraph***) is a graph whose vertex set can be divided into two disjoint sets V_1 and

V_2 such that every edge has one end in V_1 and another end in V_2. A ***complete bipartite graph*** is a bipartite graph where every vertex of V_1 is adjacent to every vertex in V_2. The complete bipartite graph with partitions of order $|V_1|=m$ and $|V_2|=n$, is denoted by $K_{m,n}$. A ***star***, denoted by $K_{1,p-1}$ is a tree with one root vertex and p – 1 pendant vertices. A ***bistar***, denoted by $B(m, n)$ is the graph obtained by joining the root vertices of the stars $K_{1,m}$ and $K_{1,n}$. The ***friendship graph***, denoted by F_n can be constructed by identifying n copies of the cycle C_3 at a common vertex. A ***wheel graph***, denoted by W_p is a graph with p vertices, formed by connecting a single vertex to all vertices of C_{p-1}. A ***helm graph***, denoted by H_n is a graph obtained from the wheel W_n by attaching a pendant vertex to each vertex in the outer cycle of W_n. ***Corona*** of two graphs G_1 and G_2, denoted by $G_1 \circ G_2$ is the *graph obtained by taking one copy of* G_1 *and* $|V_1|$ *copies of* G_2 *(*$|V_1|$ *is the number of vertices in* G_1*) in which* i^{th} *vertex of* G_1 *is joined to every vertex in the* i^{th} *copy of* G_2*. If S is a subset of V, then* $\langle S \rangle$ *denotes the vertex induced subgraph of G induced by S. The* ***open neighbourhood*** *of a set S of vertices of a graph G, denoted by N(S) is the set of all vertices adjacent to some vertex in S and* $N(S) \cup S$ *is called the* ***closed neighbourhood*** *of S, denoted by N[S]. The* ***diameter*** *of a connected graph is the maximum distance between two vertices in G and is denoted by diam(G). A* ***cut – vertex (cut edge)*** *of a graph G is a vertex (edge) whose removal increases the number of components. A* ***vertex cut****, or* ***separating set*** of a connected graph G is a set of vertices whose removal results in a disconnected graph. The ***connectivity*** or ***vertex connectivity*** of a graph G, denoted by $\kappa(G)$ (where G is not complete) is the size of a smallest vertex cut. A connected subgraph H of a connected graph G is called a **H -cut** if $(G – H) \geq 2$. The ***chromatic number*** of a graph G, denoted by $\chi(G)$ is the smallest number of colors needed to colour all the vertices of a graph G in which adjacent vertices receive different colours. For any real number , ⌊ ⌋ denotes the largest integer less than or equal to . A ***Nordhaus -Gaddum-type*** result is a (tight) lower or upper bound on the sum or product of a parameter of a graph and its complement. Terms not defined here are used in the sense of [11].

A subset S of V is called a ***dominating set*** of G if every vertex in $V - S$ is adjacent to at least one vertex in S. The ***domination number*** $\gamma(G)$ of G is the minimum cardinality taken over all dominating sets in G. A dominating set S of a connected graph G is said to be a ***connected dominating set*** of G if the induced
sub graph $<S>$ is connected. The minimum cardinality taken over all connected dominating sets is the ***connected domination number*** and is denoted by γ_c.

Many authors have introduced different types of domination parameters by imposing conditions on the dominating set [3, 16]. Recently, the concept of triple connected graphs has been introduced by Paulraj Joseph J. et. al.,[14] by considering the existence of a path containing any three vertices of G. They have studied the properties of triple connected graphs and established many results on them. A graph G is said to be ***triple connected*** if any three vertices lie on a path in

G. All paths, cycles, complete graphs and wheels are some standard examples of triple connected graphs. In [4] Mahadevan G. et. al., introduced triple connected domination number of a graph and found many results on them. A subset S of V of a nontrivial connected graph G is said to be ***triple connected dominating set***, if S is a dominating set and the induced sub graph $<S>$ is triple connected. The minimum cardinality taken over all triple connected dominating sets is called the ***triple connected domination number*** of G and is denoted by $_{tc}(G)$. In [5, 6, 7, 8, 9, 10] Mahadevan G. et. al., introduced complementary triple connected domination number, paried triple connected domination number, complementary perfect triple connected domination number, triple connected two domination number, restrained triple connected domination number, dom strong triple connected domination number of a graph. A subset S of V of a nontrivial graph connected graph G is said to be a complementary tree dominating set, if S is a dominating set and the induced sub graph $<V - S>$ is a tree. The minimum cardinality taken over all complementary tree dominating sets is called the complementary tree domination number of G and is denoted by $_{ctd}(G)$. In this paper, we use this idea to develop the concept of triple connected complementary tree dominating set and triple connected complementary tree domination number of a graph.

Theorem 1.1 [14] A connected graph G is not triple connected if and only if there exists a H -cut with $(G – H) \geq 3$ such that $|\ ()\ ()| = 1$ for at least three components C_1, C_2, and C_3 of $G – H$.

Notation 1.2 Let G be a connected graph with m vertices $v_1, v_2, \ldots, v_m$. The graph

obtained from G by attaching n_1 times a pendant vertex of on the vertex v_1, n_2

times a pendant vertex of on the vertex v_2 and so on, is denoted by $G(n_1$,

n_2 , n_3 , …., $n_m)$ *where* n_i, $l_i \geq 0$ and $1 \leq i \leq m$.

Example 1.3 Let v_1, v_2, v_3, v_4, be the vertices of C_4. The graph $C_4(P_2, 2P_2, 3P_2, P_3)$ is obtained from C_4 by attaching 1 time a pendant vertex of P_2 on v_1, 2 times a pendant vertex of P_2 on v_2, 3 times a pendant vertex of P_2 on v_3 and 1 time a pendant vertex of P_3 on v_4 and is shown in Figure 1.1.

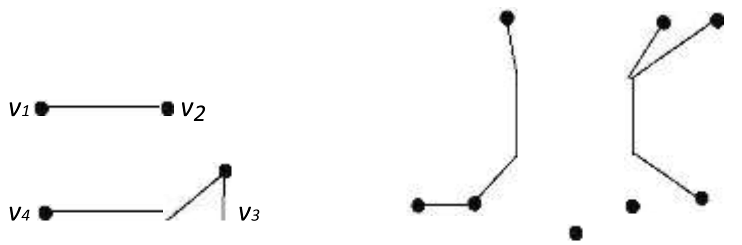


Figure 1.1 : ***$C_4(P_2, 2P_2, 3P_2, P_3)$***

2. Triple connected complementary tree domination number

Definition 2.1 A subset *S* of *V* of a nontrivial connected graph *G* is said to be a *triple connected complementary tree dominating set,* if *S* is a triple connected dominating set and the induced subgraph <*V* - *S*> is a tree. The minimum cardinality taken over all triple connected complementary tree dominating sets is called the *triple connected complementary tree domination number* of *G* and is denoted by $_{tct}(G)$. Any triple connected dominating set with $_{tct}$ vertices is called a $_{tct}$ -set of *G*.

Example 2.2 For the graph G_1 in Figure 2.1, $S = \{v_1, v_2, v_3\}$ forms a $_{tct}$ -set of G_1. Hence $_{tct}(G_1)$ = 3.

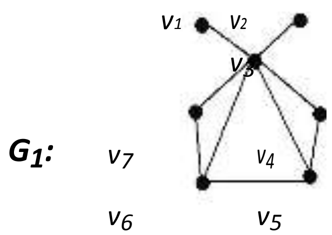


Figure 2.1 : Graph with $_{tct}$ = 3.

Observation 2.3 Triple connected complementary tree dominating set does not exists for all graphs and if exists, then $_{tct}(G) \geq 3$.
Throughout this paper we consider only connected graphs for which triple connected complementary tree dominating set exists.
Observation 2.4 The complement of the triple connected complementary tree dominating set need not be a triple connected complementary tree dominating set.
Observation 2.5 Every triple connected complementary tree dominating set is a dominating set but not conversely.
Observation 2.6 For any connected graph G, $_{c}(G) \leq _{tc}(G) \leq _{tct}(G)$ and for the cycle C_7 the bounds are sharp.
Theorem 2.7 If the induced subgraph of each connected dominating set of G has more than two pendant vertices, then G does not contain a triple connected complementary tree dominating set.
Proof The proof follows from *Theorem 1.1*.
Exact value for some standard graphs:

1) For any cycle of order $p \geq 5$, $_{tct}(C_p) = p - 2$.
2) For any complete bipartite graph of order $p \geq 5$, $_{tct}(K_{m,n}) = p - 2$.
   (where $m, n \geq 2$ and $m + n = p$).
3) For any complete graph of order $p \geq 5$, $_{tct}(K_p) = p - 2$.
4) For any wheel of order $p \geq 5$, $_{tct}(W_p) = p - 2$.

Exact value for some special graphs:

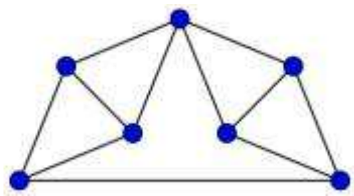


Figure 2.2

For the Moser spindle graph G, $_{tct}(G) = 3$.

2) The **Wagner graph** is a 3-regular graph with 8 vertices and 12 edges given in Figure 2.3.

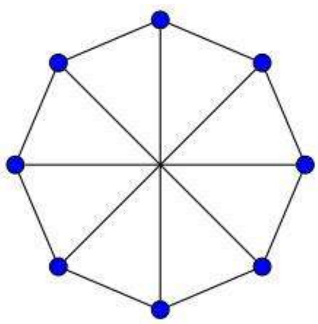


Figure 2.3

For the Wagner graph G, $_{tct}(G) = 4$.

3) The **Bidiakis cube** is a 3-regular graph with 12 vertices and 18 edges given in Figure 2.4.

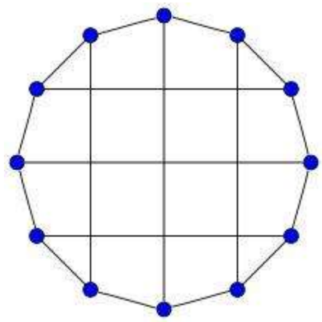


Figure 2.4

For the Bidiakis cube graph G, $_{tct}(G) = 8$.

4) The **Frucht graph** is a 3-regular graph with 12 vertices, 18 edges, and no nontrivial symmetries given in Figure 2.5.

Figure 2.5

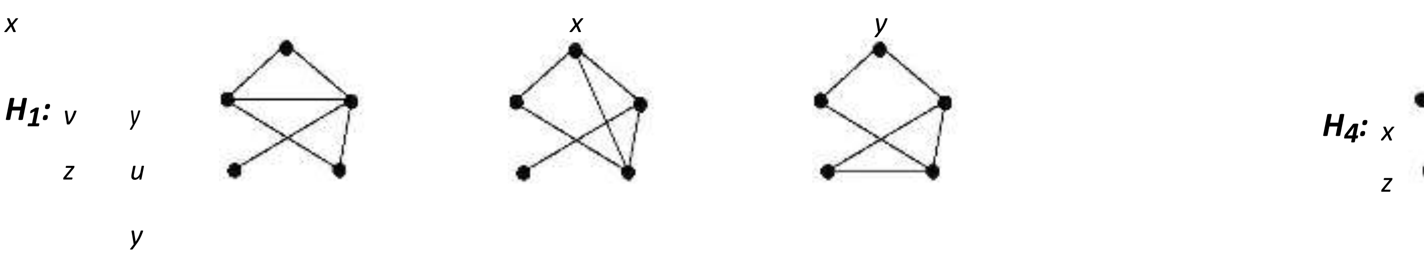
For the Frucht graph G, $_{tct}(G) = 8$.

Theorem 2.8 For any connected graph *G* with $p \geq 5$, we have $3 \leq {}_{tct}(G) \leq p - 2$

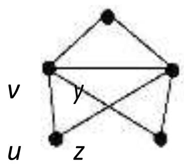
and the bounds are sharp.

Proof The lower and upper bounds follows from *Definition 2.1*. For C_5, the lower bound is attained and for K_6 the upper bound is attained.

Theorem 2.9 For a connected graph *G* with *5* vertices, $_{tct}(G) = p - 2$ if and only if *G* is isomorphic to $C_5, W_5, K_5, K_{2,3}, F_2, K_5 - \{e\}, K_4(P_2), C_4(P_2), C_3(P_3), C_3(2P_2)$ or any one of the graphs shown in Figure 2.6.



H_5:



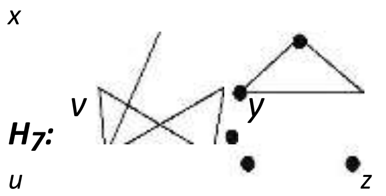


Figure 2.6 : Graphs with $_{tct} = p – 2$.

Proof Suppose G is isomorphic to C_5, W_5, K_5, $K_{2,3}$, F_2, $K_5 – \{e\}$, $K_4(P_2)$, $C_4(P_2)$, $C_3(P_3)$, $C_3(2P_2)$ or any one of the graphs H_1 to H_7 given in Figure 2.2., then clearly $_{tct}(G) = p – 2$. Conversely, let G be a connected graph with 5 vertices and $_{tct}(G) = 3$. Let $S = \{x, y, z\}$ be a $_{tct}$ -set, then clearly $<S> = P_3$ or C_3. Let

$V – S = V(G) – V(S) = \{u, v\}$, then $<V – S> = K_2$.

Case (i) $<S> = P_3 = xyz$.

Since G is connected and S is a $_{tct}$ -set, there exists a vertex say x (or z) in

P_3 which is adjacent to u and v in K_2. Then $S = \{x, y, u\}$ forms a $_{tct}$ -set of G so that $_{tct}(G) = p – 2$. If $d(x) = d(y) = 2, d(z) = 2$, then $G \cong C_5$. Since G is

connected and S is a $_{tct}$ -set, there exists a vertex say y in P_3 is adjacent to u and v in K_2. Then $S = \{x, y, z\}$ forms a $_{tct}$ -set of G so that $_{tct}(G) = p – 2$. If $d(x) = d(z) = 1, d(y) = 4$, then $G \cong C_3(2P_2)$. Now by increasing the degrees of the vertices, by the above arguments, we have G $\cong W_5, K_5, K_{2,3}$, $K_5 – \{e\}$, $K_4(P_2)$, $C_4(P_2)$, $C_3(P_3)$, and H_1 to H_7 in Figure 2.2. In all the other cases, no new graph exists.

Case (ii) $<S> = C_3 = xyzx$.

Since G is connected, there exists a vertex say x (or y, z) in C_3 is adjacent to u (or v) in K_2. Then $S = \{x, u, v\}$ forms a $_{tct}$ -set of G so that $_{tct}(G) = p – 2$. If $d(x) = 3, d(y) = d(z) = 2$, then $G \cong C_3(P_3)$. If $d(x) = 4, d(y) = d(z) = 2$, then

$G \cong F_2$. In all the other cases, no new graph exists.

Theorem 2.10 Every triple connected complementary tree dominating set must contains all the pendant vertices.

Proof Let v be a vertex of G such that d(v) = 1 and let S be a tctd – set of G. If v is not in S, then a vertex adjacent to v must be in S and hence <V – S> is disconnected, which is a contradiction.
Corollary 2.11 If G is a graph with m pendant vertices, then $_{tct}(G) \geq m$
Proof The proof is directly follows from Theorem 2.10.

The Nordhaus – Gaddum type result is given below:

Theorem 2.12 Let G be a graph such that G and have no isolates of order $p \geq 5$.

Then (i) $_{tct}(G) + {}_{tct}(\ ) \leq 2(p-2)$

2

(ii) $_{tct}(G).\ {}_{tct}(\ ) \leq 2(p-2)$ and the bound is sharp.

Proof The bound directly follows from *Theorem 2.8.* For the cycle C_7,

2

$_{tc}(G) + {}_{tc}(\ ) = 2(p-2)$ and for K_5, $_{tct}(G).\ {}_{tct}(\ ) \leq 2(p-2)$.

3 Relation with Other Graph Theoretical Parameters

Theorem 3.1 For any connected graph G with $p \geq 5$ vertices, $_{tct}(G) + \kappa(G) \leq 2p-3$ and the bound is sharp if and only if $G \cong K_p$.

Proof Let G be a connected graph with $p \geq 5$ vertices. We know that $\kappa(G) \leq p-1$

and by *Theorem 2.8,* $_{tct}(G) \leq p-2$. Hence $_{tct}(G) + \kappa(G) \leq 2p-3$. Suppose G is isomorphic to K_p. Then clearly $_{tct}(G) + \kappa(G) = 2p-3$. Conversely, Let $_{tct}(G) + \kappa(G) = 2p-3$. This is possible only if $_{tct}(G) = p-2$ and $\kappa(G) = p-1$. But $\kappa(G) = p-1$, and so $G\ K_p$.

Theorem 3.2 For any connected graph G with $p \geq 5$ vertices, $_{tct}$(G) + (G) $\leq 2p-2$ and the bound is sharp if and only if $G \cong K_p$.

Proof Let G be a connected graph with $p \geq 5$ vertices. We know that $(G) \leq p$ and by *Theorem 2.8,* $_{tct}(G) \leq p-2$. Hence $_{tct}(G) + (G) \leq 2p-2$. Suppose G is isomorphic to K_p. Then clearly $_{tct}(G) + (G) = 2p-2$. Conversely, let $_{tct}(G) + (G) = 2p-2$. This is possible only if $_{tct}(G) = p-2$ and $(G) = p$. Since $(G) = p$, G is isomorphic to K_p.

Theorem 3.3 For any connected graph G with $p \geq 5$ vertices, $_{tct}(G) + \Delta(G) \leq 2p - 3$ and the bound is sharp.

Proof Let G be a connected graph with $p \geq 5$ vertices. We know that $\Delta(G) \leq p - 1$ and by *Theorem 2.8*, $_{tct}(G) \leq p - 2$. Hence $_{tct}(G) + \Delta(G) \leq 2p - 3$. For K_8, the *bound is sharp.*

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